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By

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The Obukhov dynamic similarity of flows hypothesis for the surface boundary layer of the atmosphere has been reevaluated with respect to an arbitrary variable rather than a constant in the diabatic influence function. A literature search revealed that estimates of the so-called constant β ranged from $0.6 < \beta < 17$, suggesting that β was indeed a variable. Analysis of 103 wind and temperature profiles extracted from the literature led to the conclusion that no constant exists for the log plus linear profile form for thermally

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stratified stable flow and that the critical value of the gradient Richardson number is unity.

CONTENTS

INTRODUCTION

Richardson (1920) originally suggested that the decay of turbulence in the atmosphere, and the surface boundary layer in particular, was complete when the value of the Richardson number Ri became unity. This so-called critical Richardson number has since been estimated to occur anywhere in the range of $0.0417 \leq Ri_{crit} \leq 1.5$. A large number of the estimates of the critical Richardson number have been based upon some form of a surface boundary layer hypothesis for a thermally stratified stable regime, with Ri_{crit} traditionally inferred from the constant, β, of the diabatic influence function. There have been nearly as many estimates of the influence function constant as there have been for Ri_{crit}. A cursory review of vintage and contemporary literature reveals that $0.6 \leq \beta \leq 17$ generally reflects the range of β, the influence function constant. If $Ri_{crit} = \beta^{-1}$, then the values for Ri_{crit} tabulated in Table 1 as determined from the literature review are in good agreement with those of β given in Table II.

The above information suggests that Ri_{crit} should be near unity and that β is more likely a variable rather than a constant. The intent of this paper is to explore these postulates.

DYNAMIC SIMILARITY IN A STABLE REGIME

Obukhov (1946) proposed a dynamic similarity theory for the surface boundary layer where the depth of the surface layer was characterized by a unique length L defined as

$$L = - \frac{u_{\star} \, c_p^3 \, \rho\overline{\theta}}{kg\,H} \qquad (1)$$

TABLE I

Estimates of the Value of the
Critical Richardson Number ($0.667 \leq \beta \leq 24$)

Source	Critical Ri
Schlichting (1936) (See Schlichting 1960)	0.0417
Businger (1955)	0.105
Sverdrup (1936)	0.11
Obukhov (1946)	0.11
McVehil (1964)	0.14
Holzman (1943)	0.143
Deacon (1949)	0.15
Davis (1957)	0.15
Ellison (1957)	0.15
Lumley and Panofsky (1964)	0.14 to 0.22
Busch (1965)	0.20
Businger (1959)	0.20
Webb (1970)	0.20
Businger, et al. (1971)	0.21
Hansen and Serna (1969)	0.224
Goldstein (1931)	0.25
Ludlam (1967)	0.25
Miles (1961, 1963)	0.25
Sieffert (1958)	0.25
Taylor (1931)	0.25
Kaimal and Izumi (1965)	0.25
Webster (1964)	0.25 to 0.50
Businger (1969)	0.25 to 1.00
Townsend (1957)	0.30
Oke (1970)	0.30
Wanta (1953)	0.33 to 0.50
Portman et al. (1962)	0.35
Crawford (1965)	0.35
Rider and Robinson (1951)	0.40
Lyons, et al. (1964)	0.50
Sutton (1949)	0.50 to 1.00
Fichtl (1970)	0.55
Calder (1949)	1.0
Petterssen and Swinbank (1947)	1.0
Durst (1933)	1.0
Endlich (1964)	1.0
Kronebach (1964)	1.0
Richardson (1920)	1.0
Zavarina and Yudin (1960)	1.0
Clodman (1953)	1.5
Jaffe (1963)	1.5

TABLE II

Estimates of the Linear Influence Function Constant ($0.058 \leq Ri_{crit} \leq 1.67$)

Source	β	*β_H
Arya (1968)	10	17
Busch (1965)	5	
Businger, et al. (1971)	4.7	4.7
Chalikov (1968)	3.77 to 7.44	5.17 to 7.19
Deacon (1955)	7.1 to 12	
Deardorff (1968)	7	11
Fichtl and McVehil (1970)	7	
Gurvich (1965)	8.5	
Hansen and Serna (1969)	7	10
Holzman (1943)	7	
Lumley and Panofsky (1964)	7	
McVehil (1964)	7	
Monin and Obukhov (1954)	0.6	0.6
Oke (1970)	5	
Panofsky (1967)	10	
Panofsky, et al. (1960)	4.5	
Plate and Lin (1966)	7	
Rossby and Montgomery (1935)	11	
Sverdrup (1936)	11	
Webb (1970)	5.2	
Zilitinkevich and Chalikov (1968)	9.91	10.4

*β_H values determined from temperature profile analysis

where $u_\star$ is a friction velocity, c_p the specific heat of air at constant pressure, ρ density, $\overline{\theta}$ potential temperature, k Karman's constant, g the gravitational acceleration, and H the vertical heat flux. The scaling length L evolved from the integration of the Richardson number with respect to height as a function of the dimensionless wind shear and lapse rate. From the Obukhov hypothesis, the deduction may be made that the wind and temperature profiles in differential form are given by

$$\frac{\partial \overline{V}}{\partial z} = \frac{u_\star}{kz} \phi_M \tag{2}$$

and

$$\frac{\partial \overline{\theta}}{\partial z} = \frac{T^\star}{z} \phi_H \tag{3}$$

where $\overline{V}$ is the mean horizontal windspeed, z is height, ϕ_M and ϕ_H are the dimensionless shear and lapse rates, respectively, and $T^\star$ is a scaling temperature given by

$$T^\star = - \frac{1}{ku_\star} \frac{H}{c_p \rho} . \tag{4}$$

The eddy viscosity and eddy conductivity are then generally written as

$$K_M = ku_\star z \, \phi_M^{-1} \tag{5}$$

and

$$K_H = ku_\star z \, \phi_H^{-1} . \tag{6}$$

If, in a stable regime, the transfer of heat and momentum are assumed to be a function of mechanical turbulence only, as may be inferred from Reynolds analogy and the vertical turbulent kinetic energy budget, generally stated as

$$u_\star^2 \frac{\partial \overline{v}}{\partial z} + \frac{gH}{c_p \rho \theta} - \frac{\partial \overline{ew}}{\partial z} - \frac{\partial \overline{wp}/\rho_0}{\partial z} - \varepsilon = 0 \tag{7}$$

where $\overline{e}$ is total kinetic energy, p pressure, $\overline{w}$ the mean vertical component, and ϵ the viscous dissipation rate, then

$$K_H = K_M;\ \phi_M = \phi_H$$

and by definition

$$\frac{z}{L} = Ri\ \phi_M \tag{8}$$

where z/L is the familiar Monin and Obukhov (1954) scaling ratio. From Equation (8), L may be redefined as

$$L = \frac{u_\star \theta \partial \overline{V}/\partial z}{kg\partial\theta/\partial z}, \tag{9}$$

since

$$Ri = \frac{g\ \partial\overline{\theta}/\partial z}{\theta\ (\partial V/\partial z)^2} \tag{10}$$

with ϕ_M defined by Equation (2).

Rewriting Equation (2) as

$$\frac{\partial\overline{V}}{\partial z} = \frac{u_\star}{kz}\ \frac{z/L}{Ri} \tag{11}$$

then adding and subtracting 1 to z/L $(Ri)^{-1}$ and multiplying and dividing by z/L yields

$$\frac{\partial\overline{V}}{\partial z} = \frac{u_\star}{kz}\left[1 + \frac{z}{L}\left(\frac{z/L - Ri}{Ri\ z/L}\right)\right]. \tag{12}$$

Defining $(z/L - Ri)\ (Ri\ z/L)^{-1}$ as an arbitrary variable β, then integrating Equation (12) gives

$$\overline{V} = \frac{u_\star}{k}\left(\ln\frac{z}{z_0} + \overline{\beta}\,\frac{z}{L}\right) \tag{13}$$

where z_0 is the roughness length and $\overline{\beta}$ is the average β over a layer $z = \sqrt{z_1 z_2}$.

It is easily demonstrated that $\bar{\beta}\, z/L = \phi_M - 1$ so that Equation (13) may be stated as

$$\bar{V} = \frac{u_\star}{k}\left[\ell n\,\frac{z}{z_0} + (\phi_M - 1)\right]. \tag{14}$$

If $\phi_M = \phi_H$, then the integrated temperature profile is

$$\bar{\theta} - \bar{\theta}_0 = T^\star\left[\ell n\,\frac{z}{z_0} + (\phi_M - 1)\right] \tag{15}$$

where θ_0 is the potential temperature at z_0.

Evaluation of the profile hypothesis is dependent upon establishing L from experimental data. Equation (9) shows that the only inferred entity is the friction velocity $u_\star$. All other parameters are constant or can be easily measured. The friction velocity can be evaluated by first considering the integrated form of the wind profile [Equation (13)]. If the mean wind V_1 at z_1 is subtracted from speed V_3 at z_3, where the subscripts 1, 2, 3, ... refer to adjacent levels,

$$\Delta V_{3,1} = \frac{u_\star}{k}\left[\ell n\,\frac{z_3}{z_1} + \bar{\beta}_3\,\frac{z_3}{L} - \bar{\beta}_1\,\frac{z_1}{L}\right]. \tag{16}$$

In a geometric progression such as $y = ar^n$ with $a = 1$ and $r = 2$, the relationships between the levels may be expressed as $z_3 = 2z_2$ and $z_1 = 1/2\ z_2$. Thus, Equation (16) can be rewritten as

$$\Delta V_{3,1} = \frac{u_\star}{k}\left[\ell n\,\frac{z_3}{z_1} + \frac{z_2}{L}\left(\frac{4\bar{\beta}_3 - \bar{\beta}_1}{2}\right)\right]. \tag{17}$$

From the profile differential, the following can be determined:

$$\Delta V_{3,1} = \frac{u_\star}{k}\left[\ell n\,\frac{z_3}{z_1} + \ell n\,\frac{z_3}{z_1}\,\bar{\beta}_2\,\frac{z_2}{L}\right]. \tag{18}$$

From Equations (17) and (18)

$$\frac{4\bar{\beta}_3 - \bar{\beta}_1}{2} = \ln \frac{z_3}{z_1} \bar{\beta}_2 . \tag{19}$$

Substitution of Equation (19) in (17), after recalling that $\bar{V}_2 = u_*/k$ $[\ln z_2/z_0 + \beta_2 z_2/L]$ and that for a given profile u_*/k is constant, gives

$$\frac{\beta_2}{L} = \frac{\Delta V_{3,1} \ln \frac{z_2}{z_0} - V_2 \ln \frac{z_3}{z_1}}{z_2\left(V_2 \ln \frac{z_3}{z_1} - \Delta V_{3,1}\right)} \tag{20}$$

The friction velocity for each profile may be found from Equation (13), if the roughness length z_0 is known. For the two data samples used in the evaluation, values of z_0 were based upon those determined by Panofsky (1963).

VALIDATION OF THE HYPOTHESIS

The hypothesis was validated by using thermally stratified stable regime data extracted from the Great Plains Turbulence Program (Lettau and Davidson 1957) and Project Prairie Grass (Barad 1958). All stable profiles (Ri > 0) were plotted as $\bar{V}$, $\bar{\theta}$, = f ($\ln z$) and inspected for suspect erroneous data points. The vertical gradients were determined such that the gradients were tangent to the geometric mean heights of the layers. Of the 163 profiles available, 103 were retained for use. Friction velocities and the scaling length L were calculated for each profile. Fluctuations in the values of L with respect to height were smoothed by using an approximation of the height derivative in the form

$$\frac{\Sigma(z/L)_Z}{\Sigma z_i} = L^{-1} \qquad i = 1,2,3, \ldots \tag{21}$$

Values of the dimensionless shear were calculated from Equation (2).

The Richardson numbers for the profiles, averaged over suitable intervals, are shown as a function of z/L in Figure 1. The bars represent one standard deviation about the means. The general characteristics of the profile shape indicated that z/L as a function of Ri was parabolic. Consequently, the experimental data were fitted by least squares methods to

$$\frac{z}{L} = a + b\,Ri + c\,Ri^2 \tag{22}$$

which leads to a = 0, b = 1, and c = 15, so that

$$\frac{z}{L} = Ri + 15\,Ri^2 \tag{23}$$

The solid curve in Figure 1 represents Equation (23).

Values of $\overline{\beta}$ for each profile layer were calculated from

$$\overline{\beta} = \frac{z/L - Ri}{Ri\ z/L} \tag{24}$$

After averaging, $\overline{\beta}$ as a function of z/L and Ri was plotted as shown in Figure 2. From Equations (7) and (23),

$$\overline{\beta} = \frac{15}{1 + 15\,Ri} \tag{25}$$

and

$$\phi_M = 1 + 15\,Ri \tag{26}$$

The curve in Figure 2 represents Equation (25).

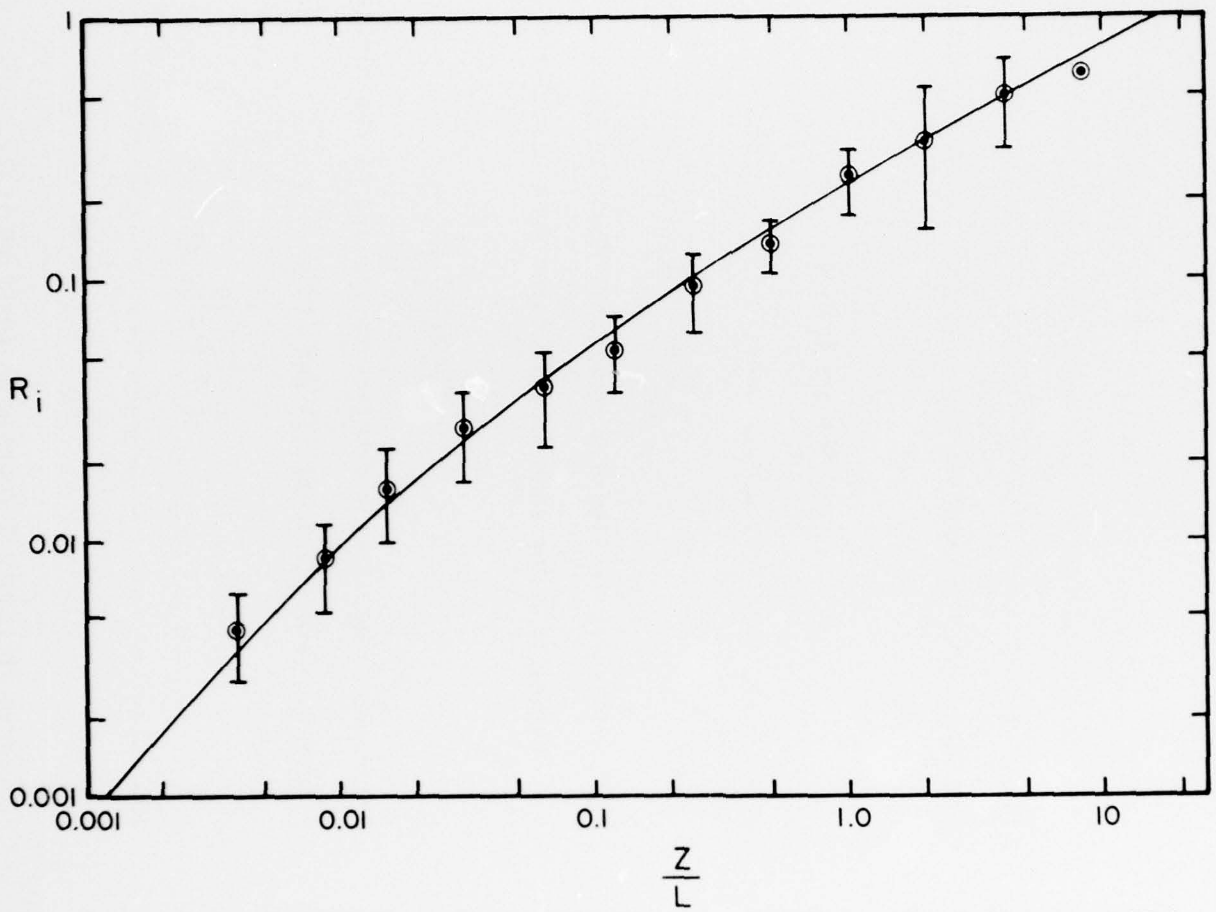

FIG. 1 THE GRADIENT RICHARDSON NUMBER AS A FUNCTION OF THE SCALING RATIO z/L.

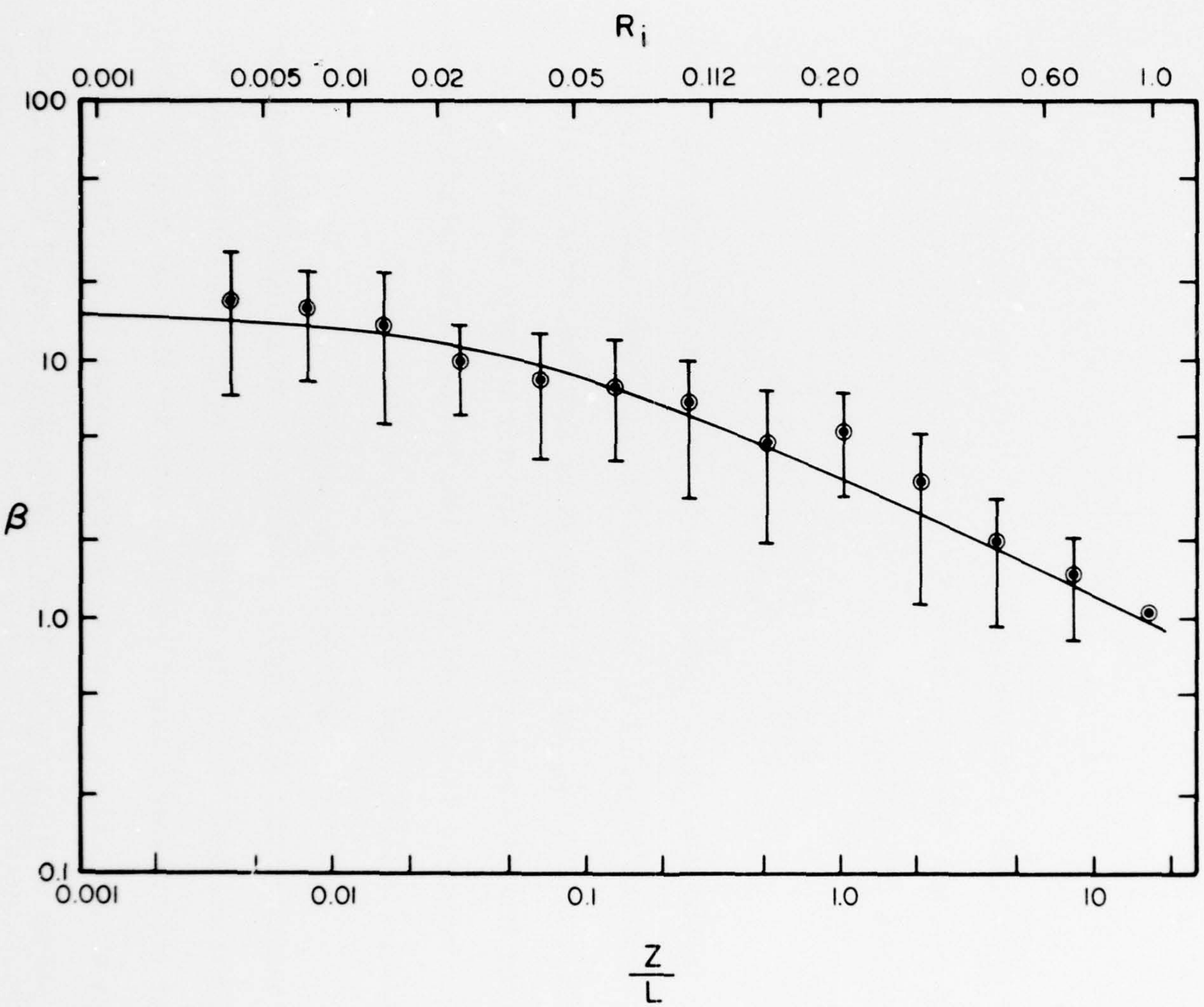

FIG. 2 β AS A FUNCTION OF z/L AND Ri

THE CRITICAL RICHARDSON NUMBER

Richardson's (1920) criterion, in terms of the production of energy can be shown to be

$$\frac{d\overline{E}(z,t)}{dt} = K_H \left(\frac{\partial \overline{V}}{\partial z}\right)^2 \left[\frac{K_M}{K_H} - \frac{g}{\theta} \frac{\partial \theta/\theta z}{(\partial V/\theta z)^2}\right] \quad (27)$$

where $d\overline{E}(z,t)/dt$ is the rate of increase of the mean turbulent kinetic energy per unit time. Factoring K_M/K_H and ϕ_M from the right-hand side of Equation (27) yields

$$\frac{dE(z,t)}{dt} = \frac{K_M}{\phi_M} \left(\frac{\partial \overline{V}}{\partial z}\right)^2 \left[\phi_M - \frac{z}{L}\right] \quad (28)$$

which implies that the complete decay of turbulence does not occur until z/L approaches the value of the concurrent ϕ_M. From Equations (23), (25), and (26), critical ϕ_M and z/L appear to be about 16, which can also be inferred from Figures 1 and 2. This strongly suggests that Ri_{crit} is unity. Values of Ri, ϕ_M, and $\overline{\beta}$ as a function of z/L as calculated with Equations (23), (25), and (26) are given in Table III.

DISCUSSION AND SUMMARY

The characteristics of thermally stratified stable flow as deduced from the Obukhov (1946) similarity theory and inferred from the vertical turbulent kinetic energy budget should account for the wide range of critical values of the gradient Richardson number reported in the literature. Many values of Ri_{crit} were evidently based upon values of $\overline{\beta}^{-1}$ calculated from data observed over an extremely narrow range of Ri or z/L. The values of $\overline{\beta}$ extracted from the literature as a function of z/L are shown in Figure 3. Again, the implication that $\overline{\beta}$ is variable is quite apparent.

TABLE III

The Richardson Number, Dimensionless Shear, and $\bar{\beta}$ as a Function of Z/L for Stable Flow.

z/L	Ri	ϕ_M	$\bar{\beta}$
0	0	1	15
0.0012	0.001	1.0150	14.78
0.0025	0.002	1.0300	14.56
0.0054	0.005	1.0750	13.95
0.0077	0.007	1.1050	13.57
0.0115	0.010	1.1500	13.04
0.0260	0.020	1.3000	11.54
0.0435	0.030	1.4500	10.34
0.0640	0.040	1.6000	9.38
0.0875	0.050	1.7500	8.57
0.10	0.055	1.8250	8.22
0.20	0.087	2.3050	6.51
0.30	0.1120	2.6800	5.60
0.40	0.1335	3.0030	5.00
0.90	0.1525	3.2875	4.56
0.60	0.1695	3.5425	4.23
0.70	0.1860	3.7900	3.96
0.80	0.2000	4.0000	3.75
0.50	0.2140	4.2100	3.56
1.0	0.2270	4.4050	3.41
2.0	0.3340	6.0100	2.50
3.0	0.4154	7.2310	2.07
4.0	0.4845	8.2675	1.81
5.0	0.5450	9.1750	1.63
6.0	0.6000	10.0000	1.50
7.0	0.6507	10.7605	1.39
8.0	0.6976	11.4640	1.31
9.0	0.7420	12.1300	1.29
10.0	0.7840	12.7600	1.18
11.0	0.8290	13.3600	1.12
12.0	0.8620	13.9300	1.08
13.0	0.8985	14.4775	1.04
14.0	0.9335	15.0025	1.00
15.0	0.9675	15.5125	0.97
16.0	1.0000	16.0000	0.94

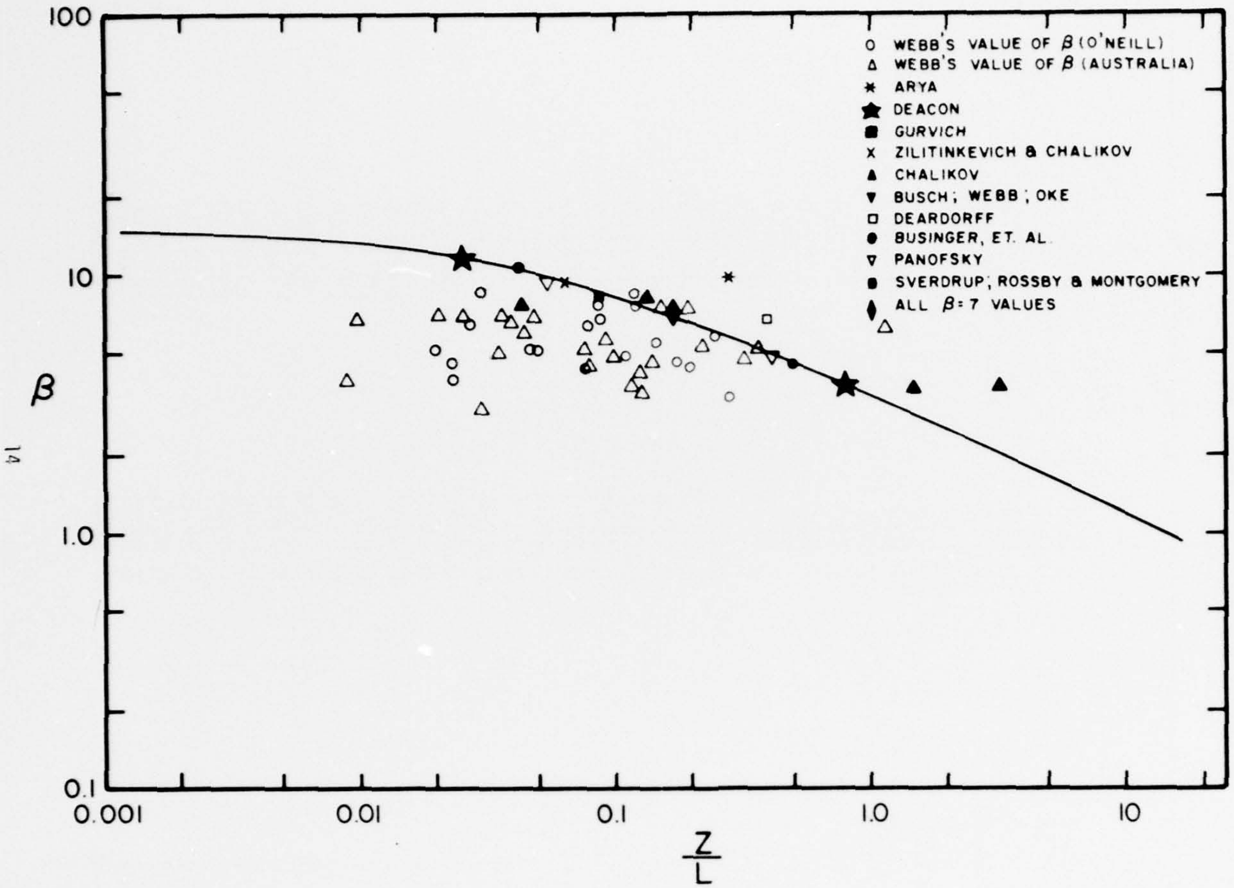

FIG. 3 β AS A FUNCTION OF z/L AS EXTRACTED FROM THE LITERATURE.

The concepts explored in this treatise are all straightforward and generally agree with the mean flow conditions observed in the surface boundary layer in thermally stratified stable air. The results are also in reasonably good agreement with the results of other experimenters, if the linear influence function and Ri_{crit} of these studies are adjusted to fit current results. The vast range of critical gradient Richardson numbers and the "constant" $\bar{\beta}$ found in the literature tend to support the author's contentions. An interesting sidelight of this study is the adiabatic value of $\bar{\beta} = 15$, which coincides with the value of $\gamma = 15$ for the linear quartic form of the unstable profile, usually written as $\phi_M = (1 - \gamma z/L)^{-1/4}$, as reported by many experimenters.

CONCLUSIONS

Two conclusions may be stated: (1) The Critical Richardson number is unity, and (2) no constant exists for the log-plus-linear influence function.

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